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A NEW FORM OF IN-PLANE TRAJECTORIES THEOREM. GENERATION WITH ROTARY CUTTERS

BY

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Abstract. The long thread machining, as driving thread of machine tools or pressing machines, is frequently made with rotary cutters. The axial section of thread is obtained by rolling, between the straight lined centrode, associated with the thread axial section, and the circular one associated with the rotary cutter.

For non-involute shapes of generated threads (in case of ogival or circle arcs profiles) a specific issue in the profiling of rotary cutter cutting edges.

The solving of this problem is made using the surface generating basic methods — Olivier or Gohman theorems, as so as the gearing basic theorem, the Willis condition.

In this paper is proposed a method developed based on the in-plane generating trajectories family, for which is developed an analytical foundation.

Key words: thread, rotary cutter, in-plane generating trajectories, surface generation, enwrapping theory.

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1. Introduction

The generation by enveloping, using the rolling method, can be solved with specific theorems (Gohman) or with complementary methods for surface enveloping study (Oancea, 2004).

The in-plane generating trajectories method may have a new interpretation based on the definition of the normals to this family.

A new analytical form of enveloping condition is developed as foundation for a new method.

2. The method of in-plane generating trajectories family

In figure 1 is presented the basic kinematics of generating with rotary cutters. Is presented the screw to be generated and its axial section, associated with a straight lined centrode, in rolling with the centrode of the rotary cutter — circle with radius R_{r2}.

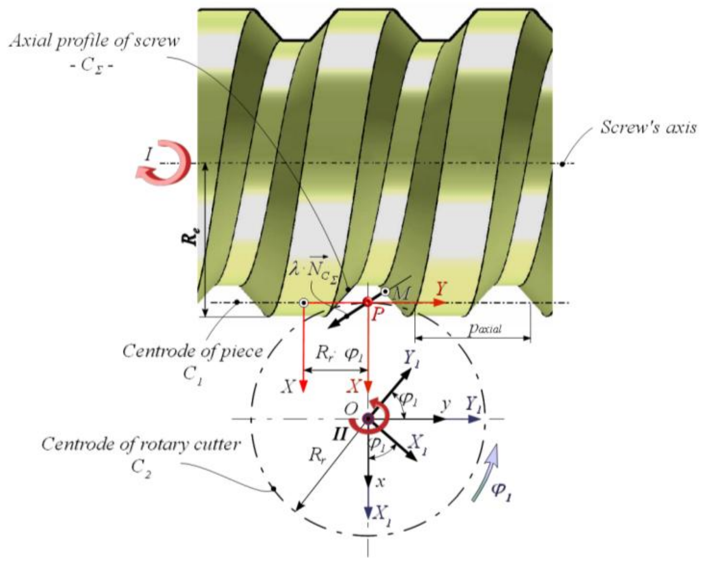

Fig. 1 − Kinematics of generation with rotary cutter; rolling centrodes

Being known the screw axial pitch — p_{axial} — it is possible to define the value of the tool's centrode radius (the rotary cutter radius) from the condition that the length of the circle with the radius R_r to be equal with an integer number of pitch.

$$2\pi \cdot R_r = K \cdot p_{axial} \quad (K \text{ integer})$$

The position of the C_1 centrode regarding the screw's external radius may be defined from the interference avoiding conditions.

They are defined the reference systems:

xy is the fixed reference system, with *O* origin. The rotation of tool (the movement *II*) has place around this origin.

X_1Y_1 — mobile reference system, joined with the C_2 centrode of the rotary cutter;

XY — mobile reference system joined with the C_1 centrode, with *Y* axis overlapped with this.

The two centrodes are tangents in the gearing pole, point *P* in figure 1.

Assuming as known the parametrical equations of the C_Σ profile, the axial section in the *XY* reference system:

$$C_\Sigma \left| \begin{array}{l} X = X(\theta); \\ Y = Y(\theta); \end{array} \right. \tag{1}$$

with θ variable parameter.

The process kinematics defines the movements:

- The absolute motion of C_1 centrode:

$$x = X + a; \quad a = \left\| \begin{array}{c} -R_r \\ -R_r \cdot \varphi_1 \end{array} \right\|, \tag{2}$$

with φ_1 movement parameter;

- The absolute motion of the C_2 centrode:

$$x = \omega_3^T(\varphi_1) \cdot X_1 . \tag{3}$$

Now, it is possible the determining of the relative movement of the screw's axial section regarding the X_1Y_1 space, associated with the rotary cutter:

$$X_1 = \omega_3^T(\varphi_1) \cdot \left[X + a \right]. \tag{4}$$

So, for the current point from the C_Σ is possible to determine the generating trajectories family, in the space associated with the rotary cutter, the X_1Y_1 space:

$$\left\| \begin{array}{c} X_1 \\ Y_1 \end{array} \right\| = \left\| \begin{array}{cc} \cos\varphi_1 & \sin\varphi_1 \\ -\sin\varphi_1 & \cos\varphi_1 \end{array} \right\| \cdot \left[\left\| \begin{array}{c} X(\theta) \\ Y(\theta) \end{array} \right\| + \left\| \begin{array}{c} -R_r \\ -R_r \cdot \varphi_1 \end{array} \right\| \right] \tag{5}$$

The normal to the C_Σ profile, see (1), is defined by

$$\vec{N}_\Sigma = \left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ \dot{X}_\theta & \dot{Y}_\theta & 0 \\ 0 & 0 & 1 \end{array} \right| = \dot{Y}_\theta \cdot \vec{i} - \dot{X}_\theta \cdot \vec{j} \tag{6}$$

Also, it is defined the normal from the current point to the C_Σ profile.

$$\vec{N}_{C_\Sigma} \left| \begin{array}{l} X = X(\theta) + \lambda \cdot \dot{Y}_\theta; \\ Y = Y(\theta) - \lambda \cdot \dot{X}_\theta, \end{array} \right. \tag{7}$$

with λ variable parameter.

The normals family to C_Σ is determined based on the transformation (4), representing the relative motion regarding the reference system joined with the C_2 centrode:

$$\left(\vec{N}_{C_\Sigma}\right)_{\varphi_1} \left\|\begin{matrix} X_1 \\ Y_1 \end{matrix}\right\| = \left\|\begin{matrix} \cos\varphi_1 & \sin\varphi_1 \\ -\sin\varphi_1 & \cos\varphi_1 \end{matrix}\right\| \cdot \left[\left\|\begin{matrix} X(\theta)+\lambda\cdot\dot{Y}_\theta \\ Y(\theta)-\lambda\cdot\dot{X}_\theta \end{matrix}\right\| + \left\|\begin{matrix} -R_r \\ -R_r\cdot\varphi_1 \end{matrix}\right\|\right] \tag{8}$$

or, after development:

$$\begin{aligned} X_1 &= \left[X(\theta)+\lambda\cdot\dot{Y}_\theta - R_r\right]\cdot\cos\varphi_1 + \left[Y(\theta)-\lambda\cdot\dot{X}_\theta - R_r\cdot\varphi_1\right]\cdot\sin\varphi_1; \\ Y_1 &= -\left[X(\theta)+\lambda\cdot\dot{Y}_\theta - R_r\right]\cdot\sin\varphi_1 + \left[Y(\theta)-\lambda\cdot\dot{X}_\theta - R_r\cdot\varphi_1\right]\cdot\cos\varphi_1. \end{aligned} \tag{9}$$

If the family of $\left(\vec{N}_{C_\Sigma}\right)_{\varphi_1}$ normals, in the rolling process between the two centrodes, follow the Willis theorem, than the screw profile — C_Σ and the profile to be determined — C_S, should be reciprocally enveloping leading to the proposed problem solving: the determination of the profile of rotary cutter cutting edge which generate by enveloping the axial section of the screw.

The coordinates of the gearing pole — P — in the X_1Y_1 reference system are, see figure 1:

$$\begin{aligned} X_1 &= -R_r\cdot\cos\varphi_1 \\ Y_1 &= R_r\cdot\sin\varphi_1 \end{aligned} \tag{10}$$

In this way, the conditions that the normals family $\left(\vec{N}_{C_\Sigma}\right)_{\varphi_1}$ pass through the gearing pole, from (9) and (10) become:

$$\begin{aligned} &\left[X(\theta)+\lambda\cdot\dot{Y}_\theta - R_r\right]\cdot\cos\varphi_1 + \left[Y(\theta)-\lambda\cdot\dot{X}_\theta - R_r\cdot\varphi_1\right]\cdot\sin\varphi_1 = -R_r\cdot\cos\varphi_1; \\ &-\left[X(\theta)+\lambda\cdot\dot{Y}_\theta - R_r\right]\cdot\sin\varphi_1 + \left[Y(\theta)-\lambda\cdot\dot{X}_\theta - R_r\cdot\varphi_1\right]\cdot\cos\varphi_1 = R_r\cdot\sin\varphi_1. \end{aligned} \tag{11}$$

Now, it is possible to determine the enveloping condition by eliminating the scalar parameter λ. By handling the equations (11) we arrive at forms:

$$\begin{aligned} &\left[X(\theta)-R_r\right]\cdot\cos\varphi_1 + \left[Y(\theta)-R_r\cdot\varphi_1\right]\cdot\sin\varphi_1 + \lambda\left[\dot{Y}_\theta\cdot\cos\varphi_1 - \dot{X}_\theta\cdot\sin\varphi_1\right] = \\ &= -R_r\cdot\cos\varphi_1; \\ &-\left[X(\theta)-R_r\right]\cdot\sin\varphi_1 + \left[Y(\theta)-R_r\cdot\varphi_1\right]\cdot\cos\varphi_1 - \lambda\left[\dot{Y}_\theta\cdot\sin\varphi_1 - \dot{X}_\theta\cdot\cos\varphi_1\right] = \\ &= R_r\cdot\sin\varphi_1. \end{aligned} \tag{12}$$

In this way, the specific enwrapping condition is determined from (12), in form:

$$\varphi_1 = \frac{X\cdot\dot{X} + Y\cdot\dot{Y}}{-R_r\cdot\dot{Y}}. \tag{13}$$

Basically, the enwrapping condition (13) is a dependency between the variable parameters θ and φ_1, in principle in form:

$$\theta = \theta(\varphi_1) \tag{14}$$

Associating the condition (13) to the in-plane generating trajectories family, see (5) or (9), for λ=0,

$$\left(T_{C_\Sigma}\right)_{\varphi_1} \begin{vmatrix} X_1 = \left[X(\theta) - R_r\right] \cdot \cos\varphi_1 + \left[Y(\theta) + R_r \cdot \varphi_1\right] \cdot \sin\varphi_1; \\ Y_1 = -\left[X(\theta) - R_r\right] \cdot \sin\varphi_1 + \left[Y(\theta) + R_r \cdot \varphi_1\right] \cdot \cos\varphi_1. \end{vmatrix} \tag{15}$$

the profile of rotary cutter, C_S, is determined as enveloping of trajectories family, $\left(T_{C_\Sigma}\right)_{\varphi_1}$.

3. Numeric application. Rotary cutter for trapezoidal thread

The axial profile of the trapezoidal thread is presented in figure 2. Here are also presented the rolling centrodes and the reference systems.

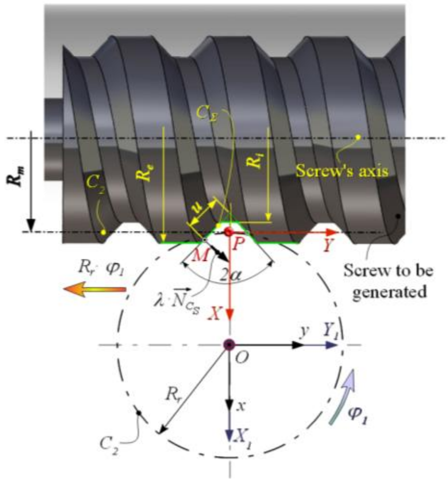

Fig. 2 − Axial profile of the trapezoidal thread

The reference systems are defined, see also figure 1:
xy is the global reference system, with O origin as centre of the C_2 centrode;
X_1Y_1 — mobile reference system joined with the rotary cutter and the C_2 centrode; XY — mobile reference system joined with the axial section of the screw to be generated.

The C_Σ thread profile is described by equations:

$$C_\Sigma \begin{vmatrix} X = -\dfrac{p}{4 \cdot \tan\alpha} + u \cdot \cos\alpha; \\ Y = -u \cdot \sin\alpha. \end{vmatrix} \tag{16}$$

with u variable parameter and b constructive value.

The normal direction to C_Σ is calculated in vectorial form:

$$\vec{N}_{C_\Sigma}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \cos\alpha & -\sin\alpha & 0\\ 0 & 0 & 1\end{vmatrix}=-\sin\alpha\cdot\vec{i}-\cos\alpha\cdot\vec{j}\,. \tag{17}$$

Assuming as known the directrix parameters of the $\vec{N}_{C_\Sigma}$ normal direction, is possible to write the equations of this normal in the point $M(X,Y)$ onto the curve C_Σ:

$$\vec{N}_{C_\Sigma}=\left(-\frac{p}{4\cdot\tan\alpha}+u\cdot\cos\alpha-\lambda\cdot\sin\alpha\right)\cdot\vec{i}+\left(-u\cdot\sin\alpha-\lambda\cdot\cos\alpha\right)\cdot\vec{j}\,. \tag{18}$$

So, the normals family, $\left(\vec{N}_{C_\Sigma}\right)$, in its relative motion regarding the rotary cutter, is give by transformation (4):

$$\left(\vec{N}_{C_\Sigma}\right)_{\varphi_1}\left|\begin{aligned}&X_1=-\frac{p\cdot\cos\varphi_1}{4\cdot\tan\alpha}+u\cdot\cos\left(\alpha+\varphi_1\right)-\lambda\cdot\sin\left(\alpha+\varphi_1\right)-\\&-R_r\cdot\cos\varphi_1-R_r\cdot\varphi_1\cdot\sin\varphi_1;\\&Y_1=\frac{p\cdot\sin\varphi_1}{4\cdot\tan\alpha}-u\cdot\sin\left(\alpha+\varphi_1\right)-\lambda\cdot\cos\left(\alpha+\varphi_1\right)+\\&+R_r\cdot\sin\varphi_1-R_r\cdot\varphi_1\cdot\cos\varphi_1.\end{aligned}\right. \tag{19}$$

For λ=0, the normals family (19) reduces at the in-plane generating trajectories family of current points onto C_Σ, regarding the X_1Y_1 reference system:

$$\left(T_{(u)}\right)_{\varphi_1}\left|\begin{aligned}&X_1=-\frac{p\cdot\cos\varphi_1}{4\cdot\tan\alpha}+u\cdot\cos\left(\alpha+\varphi_1\right)-R_r\cdot\cos\varphi_1-R_r\cdot\varphi_1\cdot\sin\varphi_1;\\&Y_1=\frac{p\cdot\sin\varphi_1}{4\cdot\tan\alpha}-u\cdot\sin\left(\alpha+\varphi_1\right)+R_r\cdot\sin\varphi_1-R_r\cdot\varphi_1\cdot\cos\varphi_1.\end{aligned}\right. \tag{20}$$

The enveloping of this curves family from space X_1Y_1 represents the profile of the rotary cutter if is associated with the specific enwrapping condition. The enwrapping condition is the condition that the C_Σ normals family pass through the gearing pole.

In the X_1Y_1 reference system, the gearing pole P (tangency point of the two conjugated centrodes, C_1 and C_2) has the coordinates:

$$P\left|\begin{aligned}&X_1=-R_r\cdot\cos\varphi_1;\\&Y_1=R_r\cdot\sin\varphi_1.\end{aligned}\right. \tag{21}$$

From (19) and (21) results:

$$\begin{aligned}&X_1=-\frac{p\cdot\cos\varphi_1}{4\cdot\tan\alpha}+u\cdot\cos\left(\alpha+\varphi_1\right)-\lambda\cdot\sin\left(\alpha+\varphi_1\right)-\\&-R_r\cdot\cos\varphi_1-R_r\cdot\varphi_1\cdot\sin\varphi_1=-R_r\cdot\cos\varphi_1;\\&Y_1=\frac{p\cdot\sin\varphi_1}{4\cdot\tan\alpha}-u\cdot\sin\left(\alpha+\varphi_1\right)-\lambda\cdot\cos\left(\alpha+\varphi_1\right)+\\&+R_r\cdot\sin\varphi_1-R_r\cdot\varphi_1\cdot\cos\varphi_1=R_r\cdot\sin\varphi_1.\end{aligned} \tag{22}$$

The λ parameter is eliminated from the equations assembly (22), obtaining the enwrapping condition:

$$\varphi_1 = \frac{-u + a \cdot \cos\alpha}{R_r \cdot \sin\alpha}. \tag{23}$$

The parameter *u* varies between the limits:

$$u_{\min} = \frac{\dfrac{p}{4 \cdot \tan\alpha} - \left(R_m - R_i\right)}{\cos\alpha}; \quad u_{\max} = \frac{\dfrac{p}{4 \cdot \tan\alpha} + \left(R_e - R_m\right)}{\cos\alpha}; \; R_m = \frac{\left(R_e + R_i\right)}{2}. \tag{24}$$

Numerical application. A numerical application is presented for a rotary cutter with dimensions: R_e=39 mm, R_i=50 mm, axial pitch, p_{axial}=18.849 mm, rolling radius of the rotary cutter R_r=75 mm, α=15°.

The coordinates and shape of the rotary cutter flank are presented in table 1 and figure 3.

Table 1

Coordinates of the rotary cutter profile

Crt. no.	X_1 [mm]	Y_1 [mm]	Crt. no.	X_1 [mm]	Y_1 [mm]
1	-83.060	-1.592	6	-74.268	-4.817
2	-80.824	-2.701	7	-73.319	-4.947
3	-78.805	-3.544	8	-72.671	-4.993
4	-77.025	-4.153	9	-72.329	-4.996
5	-75.508	-4.566	10	-72.295	-4.995

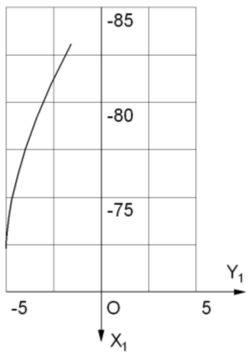

Fig. 3 − The profile of the rotary cutter.

4. Conclusions

The method of in-plane generating trajectories represents a complementary method for study of enveloping surfaces associated with a pair of rolling centrodes.

The new form of enveloping condition is simple and intuitive. The numerical application proves the method's quality.

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O NOUĂ FORMA A TEOREMEI TRAIECTORIILOR PLANE. GENERAREA CU CUŢITE ROTATIVE

(Rezumat)

Prelucrarea filetelor lungi, cum ar fi şuruburi conducătoare din construcţia maşinilor-unelte sau a preselor, se realizează frecvent cu scule de tip cuţit rotativ. Generarea secţiunii axiale a şuruburilor se face printr-un proces de generare prin rulare, în procesul de rostogolire fără alunecare între centroida rectilinie, asociată secţiunii axiale a şurubului şi centroida circulară, asociată cuţitului rotativ.

Pentru formele neevolventice ale şuruburilor generate, o problemă specifică o constituie profilarea muchiilor active ale cuţitului rotativ.

Rezolvarea problemei se face apelând la metodele fundamentale ale generării suprafeţelor prin înfăşurare – teoremele Olivier sau Gohman, precum şi la teorema fundamentală a angrenării, teorema Willis (Litvin, 1984; Radzevich, 2008; Oancea, 2004).

În lucrare, se propune o metodică, dezvoltată în baza familiei de traiectorii plane de generare, pentru care se dezvoltă un suport analitic specific, (Teodor *et al*., 2014; Berbinschi *et al.,* 2010) şi, de asemenea, se prezintă o aplicaţie numerică.